THE FERRITE-LOADED WAVEGUIDE DISCONTINUITY PROBLEM

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Abstract

A method to obtain the scattering matrix of a waveguide inhomogeneously loaded with ferrite is presented. Numerical results are obtained for the dielectric-loaded twin-slab phase shifter in rectangular waveguide.

Introduction

Discontinuity problems in anisotropically-loaded waveguides are important from both the theoretical and practical point of view (impedance matching, accurate measurement of ferrite parameters, etc.). As in the isotropic case, two types of problems are considered:

1. Single aperture discontinuity: a waveguide is isotropically loaded for $z < 0$, and totally or partially loaded with anisotropic media for $z > 0$. The plane $z = 0$ is the aperture plane.

2. Double aperture discontinuities: a finite length of waveguide is totally or partially loaded by anisotropic media; the rest of the waveguide is homogeneously or inhomogeneously loaded by isotropic media. The two aperture planes coincide with the ends of the anisotropic section of the waveguide.

In both cases, the objective is to solve the field problem and to obtain the scattering matrix describing the discontinuity. The formulation is restricted to the class of problems in which the ferrite section (the part of the waveguide loaded with ferrite) satisfies the following requirements:

1. The applied H_{dc} field is transverse to the direction of propagation (in rectangular waveguides, normal to the broad walls of the guide, and in circular and coaxial guides, in the azimuthal or $\emptyset$ direction).

2. The only modes considered are TE_{no} modes; i.e., those with only one component of electric field which is also coincident with the direction of the applied H_{dc} field and, furthermore, with all field components showing no variation along the direction of the applied H_{dc} field.

3. The bounding surfaces of the dielectrics and ferrites loading the waveguide are parallel to the applied H_{dc} field. This ensures that only the modes of the TE_{no} set are excited.

The literature dealing with discontinuity problems in anisotropically loaded waveguides is sparse. Apart from early work on small discontinuities and general discussions of the subject, the first attempt at a solution was made by Suhl and Walker[1]. An explicit formulation of the mode-matching method was obtained by Epstein[2], and solved by Sharpe and Heim[3], who found a series solution for the aperture electric field and the equivalent circuit reactance. Shortly thereafter, Lewin[4] found a closed-form solution and pointed out a paradoxical result. Lewin's paradox attracted a number of researchers; for a summary of their results and the still-standing problems, consult [5]. Bresler[6] formulated a general approach to the single-discontinuity problem, leading to an integral equation which was solved by variational methods. His method constitutes the only general systematic approach published to date. He also considered the double-discontinuity problem when the length of the anisotropic section is large enough to neglect the coupling between apertures due to beyond-cutoff modes.

The Single-Aperture Discontinuity Problem

For the class of problems under consideration, only modes of the TE_{no} (Resp. TE_{on}) set are needed. The boundary conditions at the aperture plane (continuity of tangential E and H fields) are written (mode-matching), with the amplitude of the incident TE_{10} (resp. TE_{01}) mode equal to 1. The standard method used in isotropic problems fails because the E_n and H_n fields in the ferrite section are not orthogonal sets, and the H fields are not simply related to the E fields by means of a scalar admittance.

Bresler[6] used a "bi-orthogonal" relationship between E and H to obtain an integral equation involving both the aperture electric and magnetic fields. We follow here a somewhat simpler formulation leading to an integral equation in which the only unknown is the aperture electric field $\mathcal{E}(s)$. This method requires perhaps more numerical work but has the advantage that the ferrite losses can be readily included. There are several ways to circumvent the problem arising from the non-orthogonality of the ferrite fields. For example, the fields can be orthogonalized by the Gram-Schmidt method; this would, however, introduce a set of new functions which do not correspond in general to any identifiable physical mode. It was decided to try first to expand the ferrite modes in terms of an orthogonal set of modes, called the EDG set (Equivalent Dielectric Guide); these are the modes supported by the ferrite section in the dielectric limit. The main advantage of this expansion is that it leads naturally to the "dielectric approximation," in which the higher-order modes in the ferrite section are replaced by the higher-order modes in the dielectric waveguide (the higher-order modes in the ferrite section have in general complex propagation factors, and their determination is not a trivial problem). When the ferrite fields are expanded in the prescribed manner, an integral equation involving $\mathcal{E}(s)$ is obtained. The complexity of the kernel indicates that variational methods are appropriate. The generalized Ritz-Rayleigh method is used, in connection with the Schwinger-Levine stationary principle, to obtain a linear system of equations, the unknowns being the amplitudes of the reflected modes. (It must be noted that due to the non-hermitian nature of the kernel, stationary rather than extremum principles have to be used). The number of equations needed for obtaining the coefficients is not known "a priory;" the usual procedure of increasing this number until the differential error becomes negligible can be used. Also, the solution can be checked in two different ways:

1. The scattering matrix has to be of a certain form. Heller[7] has shown that the scattering matrix of a 2-port junction of the type considered here has the form

$$\begin{pmatrix} a\exp(j\alpha) & \sqrt{1-a^2}\exp[j(\alpha-\phi)] \\ -\sqrt{1-a^2}\exp[j(\delta+\phi)] & a\exp(j\delta) \end{pmatrix}$$

where a, α, ϕ, δ are real numbers.

2. The aperture field (in general complex) can be computed as a sum of modes from both sides of the junction, at a selected grid of points in the aperture plane. Both the real and imaginary part have to match.

The method developed in the previous sections has been applied to the junction between an empty rectangular waveguide and one loaded from $z = 0$ to $z = \infty$ with two symmetrically placed slabs of lossless ferrite magnetized at remanence in opposite directions and with dielectric loading between (Fig. 1). This configuration is the widely used theoretical model of the dielectric loaded twin-slab remanence latching phase shifter[8]. The results appearing in Fig. 3 show the absolute value of the reflection coefficient and the estimated error (obtained from the energy residual), as a function of normalized remanent magnetization. The broken line corresponds to the case where only the TE_{10} mode in the ferrite section is used, all the others being replaced by the dielectric modes. In view of the relatively large error, the approximation was improved by retaining the TE_{10} and TE_{20} modes of the ferrite section, the rest being replaced by the dielectric modes as before. There is a marked reduction in error, but the results suggest that the validity of the dielectric approximation is limited. This was taken into consideration in the solution of the double-aperture discontinuity problem.

The Double-Aperture Discontinuity Problem

We now assume that the anisotropic section has a finite length $L = 2$ (Fig. 3). Using again the standard mode-matching procedure and expanding the ferrite fields in terms of the fields propagated by the isotropic section (instead of the EDG set), a system of coupled equations is obtained, from which a linear system of equations can be derived. The system was solved for the same geometry. Since the dielectric approximation is not used, the propagation factors of the higher-order ferrite modes were obtained numerically. Typically, each of these could be obtained in approximately 15 seconds of computer time (IBM 360, FORTRAN IV). Solution of the 20 x 20 system of equations (with complex coefficients) would then take about 60 seconds for each set of parameters. The results appear in Fig. 4, which shows the elements of the scattering matrix as a function of the normalized length of the ferrite section, L/λ_o (λ_o = free space wavelength). Throughout the computations, the normalized remanent magnetization was fixed at 0.5. The other scales in Fig. 4 show the differential phase shift (proportional to the length) and the magnitudes L/λ^+, L/λ^-, L/λ_{av}, where λ_{av} is $(\lambda^+ + \lambda^-)/2$. Fig. 4 shows that the minima of "a" (absolute value of the reflection coefficient) are very sharp; their location can be predicted fairly accurately because they fall near the values of $L/\lambda_{av} = n/2$, with $n = 1, 2, \ldots,$. Note also that if a given value of the differential phase shift is desired, it might be coupled with a large value of "a". By altering the loading parameters of the device, the valleys of "a" might be made coincident with at least some of the desired values of the phase shift.

Fig. 5 shows a plot of the normalized input impedance Z_i/Z_1 as a function of the normalized length L/λ_o, where Z_i is the actual impedance seen at $z = -L/2$ looking in the +z direction. For $L = 0$ the graph starts at the center (perfect match). As L/λ_o increases in intervals of 0.02 (numbered black dots) the graph describes a loop and goes back near the center when $L \simeq \lambda_{av}/2$. After 3 loops, the influence at one aperture of the higher order modes excited at the other has practically disappeared, and the graph repeats itself. The hollow dots in Fig. 7 correspond to the dielectric limit, when $\omega'_m = 0$. Note that they fall near the corresponding black dots, because with $\omega'_m = 0.5$ the ferrite fields do not depart drastically from the fields that obtain in the dielectric limit.

Conclusions

Waveguide discontinuity problems involving finite or infinite sections of transversely magnetized ferrites are solved by a mode-matching procedure which leads to a linear system of equations. This system can be solved with good accuracy by truncation, at a reasonable matrix-size. The method can be used to obtain the scattering matrix of the junction, and to obtain the design parameters for an impedance matching network. The dielectric approximation can be used (in conjunction with expansion of the ferrite fields in terms of the EDG fields) when computing time is to be minimized, at the price of less accuracy.

References

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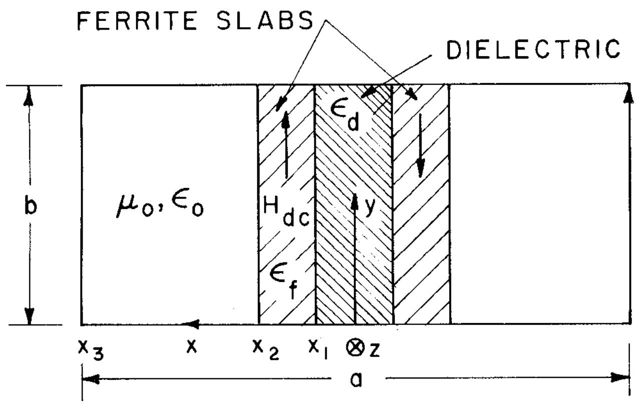

Figure 1

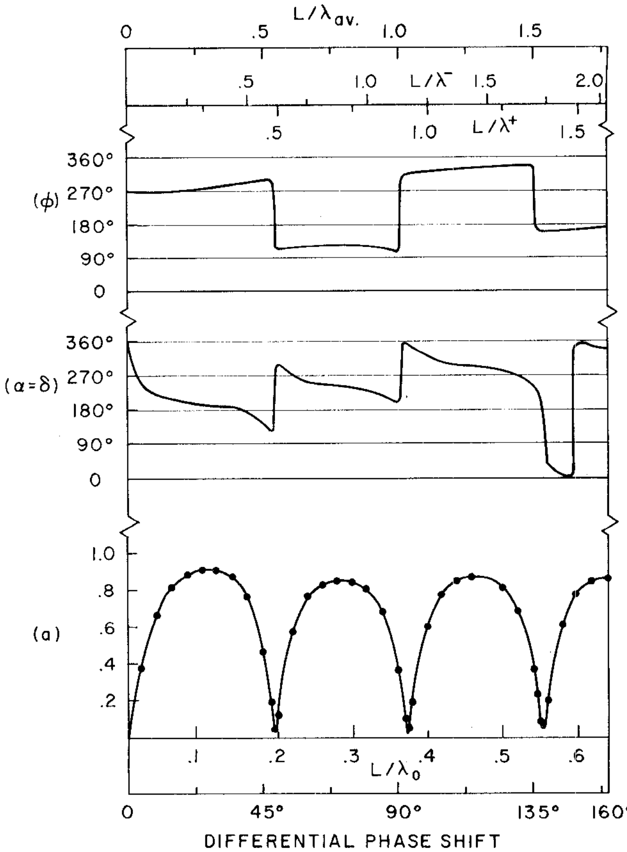

Figure 4

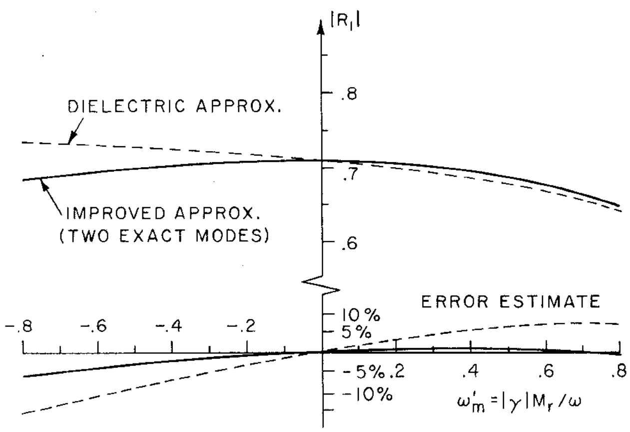

Figure 2

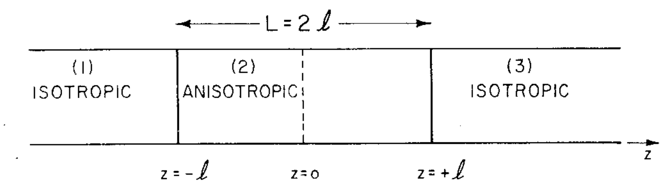

Figure 3

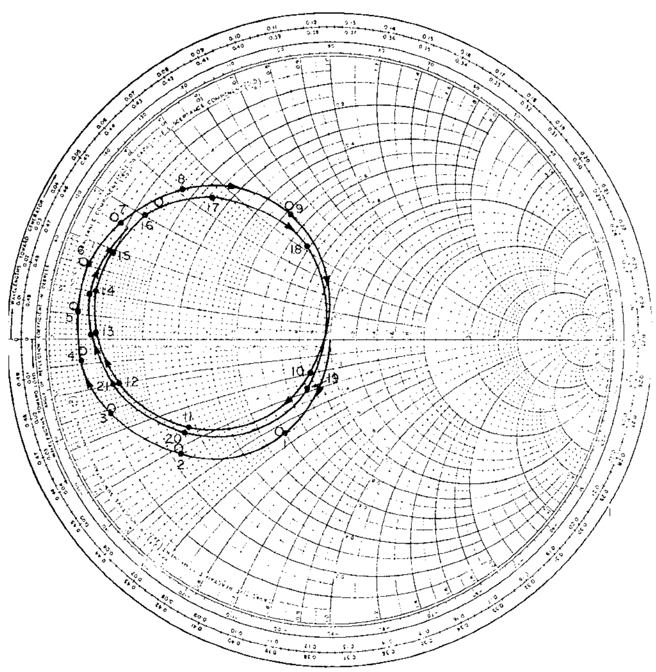

Figure 5